

3 Yr. Degree/4 Yr. Honours 6th Semester Examination, 2026 (CCFUP)

Subject : Mathematics

Course : MATH 6014 (MAJOR)

(Mechanics)

Full Marks: 60

Time: 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

2×10=20

1. Answer any ten questions:

- (a) Write down the conditions of equilibrium of a system of non-coplanar forces.
- (b) For a system of coplanar forces, explain the concept of 'Astatic centre' geometrically.
- (c) State the principle of virtual work for a particle.
- (d) A particle is executing a simple harmonic motion such that its period of oscillation is π seconds. If its maximum acceleration is 12ft/sec, find its amplitude.
- (e) Prove that a central orbit is a plane curve.
- (f) State Kepler's law of planetary motion.
- (g) Define Poinsot's central axis in a system of forces acting in a rigid body.
- (h) Prove that at an apse, the particle moves at right angle to the radius vector.
- (i) What is catenary of uniform strength? Write down its Cartesian equation. 1+1
- (j) The speed v of a particle moving along the x -axis is given by $v^2 = n^2(8bx - x^2 - 12b^2)$, where b, n are constants. Show that the motion is simple harmonic, with its centre at $x = 4b$ and of amplitude $2b$. 1+1
- (k) Prove that if a particle moves along a central orbit, then the angular momentum of the particle about the centre of force, at any instant is always a constant.
- (l) Write down the inertia matrix at a point of a rigid body with respect to the principal axes. What do the diagonal elements of the matrix represent? 1+1
- (m) State d'Alembert's principle.
- (n) Show that, for a mass of fluid in equilibrium, the surfaces of equipressure cut the lines of force at right angles.
- (o) Show that the free surface of a homogeneous liquid at rest under gravity is horizontal.

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2. Answer any four questions:

- (a) Define a common catenary. Deduce the cartesian equation of a common catenary. 5×4=20
1+4
- (b) State the laws of limiting friction. A uniform heavy elliptic wire, whose semi-axes a and b , is hung over a small rough peg. Show that if the wire can be in equilibrium with any point of it in contact with the peg, the coefficient of friction must not be less than $\frac{a^2 - b^2}{2ab}$.
- (c) A regular hexagon ABCDEF consists of equal uniform rods each of weight ' w ' freely joined together. The hexagon rests in a vertical plane and AB is in contact with a horizontal table. If C and F be connected by a light string, prove that the tension is $w\sqrt{3}$.
- (d) A force P acts along the axis of x and another force nP along a generator of the right circular cylinder $x^2 + y^2 = a^2$. Show that the Poinsot's central axis lies on the cylinder:

$$n^2(nx - z)^2 + (1 + n^2)^2 y^2 = n^4 a^2$$
- (e) If k_1 and k_2 be the radii of gyration of an elliptic lamina about two conjugate diameters, then prove that, $\frac{1}{k_1^2} + \frac{1}{k_2^2} = 4\left(\frac{1}{a^2} + \frac{1}{b^2}\right)$.
- (f) A given volume V of heavy liquid is in equilibrium under the action of body forces $-\mu x, -\mu y, -\mu z$, whose components act along the coordinate axes (μ is a constant). Find the equation of the free surface. 10×2=20

3. Answer any two questions:

- (a) (i) A uniform lamina in the form of an equilateral triangle ABC of side $\frac{4a}{\sqrt{3}}$ unit rests in equilibrium with BC on the top of a fixed sphere of radius a . What is the greatest weight which can be attached to the vertex A so that the equilibrium may remain stable?
- (ii) Find the centre of gravity of solid sphere of radius ' a ' cut off by a plane at a distance ' c ' from the centre. Hence find the centre of gravity of the solid hemisphere of radius ' a '. 5+(4+1)
- (b) (i) ABC is a triangular lamina with the side AB in the surface of a heavy homogeneous liquid. A point D is taken in AC, such that the thrusts on the areas ABD and DBC are equal. Then prove that $AD : AC = 1 : \sqrt{2}$.
- (ii) A particle of unit mass is projected with velocity u at an inclination α above the horizontal in a medium, the resistance of which is k times the velocity. Show that its direction will again make an angle α with horizon after a time $\frac{1}{k} \log_e \left(1 + \frac{2ku}{g} \sin \alpha\right)$. 5+5
- (c) (i) Derive a necessary and sufficient condition for equilibrium of a fluid under the action of a force whose components are X, Y, Z along the axes.
- (ii) A given volume V of liquid is acted upon by forces $-\frac{\mu x}{a^2}, -\frac{\mu y}{b^2}, -\frac{\mu z}{c^2}$ ($\mu > 0$, a constant). 5+5

Find the equation of free surface.

- (d) (i) A uniform rod OA of length $2a$ free to turn about its one end, revolves with uniform angular velocity ω about the vertical axis OZ through O and is inclined through a constant angle α to OZ. Show that the value of α is either 0 or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$, where $3g < 4a\omega^2$. (3)
- (ii) Prove that if the forces per unit mass at (x, y, z) parallel to the co-ordinate axes are $y(a - z), x(a - z), xy$, the surfaces of equal pressures are hyperbolic paraboloids and the curves of equal pressure and density are rectangular hyperbolas. 5+(2+3)

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